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ABSTRACT

The purpose of this paper is to illustrate the use of tobit analysis and tobit decomposition in educational research. Tobit estimates of growth rate in the presence of a ceiling effect were compared with ordinary least squares (OLS) and weighted least squares (WLS) estimates. The tobit estimates had the smallest standard error, the smallest bias, and the smallest mean squared error. (Author)

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TOBIT REGRESSION: HOW TO GET BETTER ESTIMATES WHEN
THERE ARE FLOOR OR CEILING EFFECTS

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Abstract

The purpose of this paper is to illustrate the use of tobit analysis and tobit decomposition in educational research. Tobit estimates of growth rate in the presence of a ceiling effect were compared with OLS and WLS estimates. The tobit estimates had the smallest standard error, the smallest bias and the smallest MSE.

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TOBIT REGRESSION: HOW TO GET BETTER ESTIMATES WHEN THERE ARE FLOOR OR CEILING EFFECTS

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Objective

The purpose of this paper is to illustrate the use of tobit analysis and decomposition in educational research.

Perspective

It is not unusual in educational research to encounter variables which have many observations clustering around an upper or lower limit. For example, test and attitude survey scores are subject to both floor and ceiling effects and data on school vandalism and crime tend to have many zero values. When ordinary least squares regressions are performed on such variables, violated assumptions can lead to large standard errors of estimate. Data transformations and the use of weighted least squares might help somewhat, but problems still remain.

Another alternative is tobit analysis (Tobin, 1958), a technique that has its origin in econometric research. Tobit analysis is a regression model which can be applied when the dependent variable (e.g., test score) is subject to an upper or lower limit. In the case of a ceiling effect the model is

$$Y_t = Y^* \text{ if } X\beta_1 + \beta_0 + \epsilon > Y^* \\ = X\beta_1 + \beta_0 + \epsilon \text{ otherwise}$$

where Y is the obtained score, Y^* is the maximum possible score and ϵ is a normally distributed error term with mean zero and variance σ^2 . For the purpose of this discussion, we will assume that Y is a test score that varies with age (X), and that we are trying to estimate growth rate. The expected value of Y is given by

$$E(Y|Z) = Y^* + \sigma z[1 - F(z)] - \sigma f(z) \quad (1)$$

where $z = (\beta_1 X + \beta_0 - Y^*)/\sigma$, β_1 and β_0 are the tobit regression

coefficient and constant, σ is the standard error of the tobit regression, and $f(z)$ and $F(z)$ are the standard normal density and distribution functions, respectively. The overall growth rate and the growth rate for subjects who score below the ceiling of the test can both be estimated (Mathematica Policy Review, n.d.):

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$$\partial E(Y)/\partial X = \beta_1 [1 - F(z)] \quad (2)$$

and

$$\partial E(Y^*)/\partial X = \beta_1 \left\{ 1 + \frac{zf(z)}{[1-F(z)]} + \frac{f(z)^2}{[1-F(z)]^2} \right\} \quad (3)$$

Additional information can be provided through a decomposition of the change in test score with respect to age (overall growth rate) into two parts: (1) the change in test score of those below the ceiling weighted by the probability of being below the ceiling; and (2) the change in the probability of being below the ceiling weighted by the expected value of the test score for subjects below the ceiling (McDonald & Moffitt, 1980). This decomposition makes it possible for one to see the extent to which estimates of growth rate are affected by ceiling and floor effects. McDonald and Moffitt derived the composition for the case of the lower limit. When there is an upper limit, the decomposition becomes

$$\partial E(Y)/\partial X = [1 - F(z)] \partial E(Y^*)/\partial X + E(Y^*) \partial [1 - F(z)]/\partial X \quad (4)$$

McDonald and Moffitt also calculate the fraction of the change in $E(Y)$ due to the effect above a lower limit is equal to $1 - zf(z) - f(z)^2 / F(z)^2$.

For an upper limit the fraction of the total effect $\partial E(y) / \partial E(x)$ due to observations below the upper limit would be

$$1 + zf(z)/[1 - F(z)] + f(z)^2 / [1 - F(z)]^2 \quad (5)$$

This is the coefficient of β_1 in Equation 3.

Problem

The value-added model for preschool program evaluation (Bryk & Weisberg, 1976) uses a regression of test score on age to obtain no-treatment expectations. Using the growth rate obtained from that regression, an expected posttest score is calculated based on the student's age at the time of the posttest. The treatment effect (called the value-added) is obtained by subtracting the expected posttest score from the actual posttest score. This study explores the effect of using tobit analysis to estimate growth rate when the underlying growth function exhibits nonlinearity due to ceiling effects. The hypothesis is that use of tobit analysis will provide more accurate treatment effect estimates than either ordinary or weighted least squares.

Method

A completely crossed design was used with sample size assuming two values, 30 and 120. The size of the mean actual value-added took on one of three levels, $.1\sigma$, $.25\sigma$, or $.5\sigma$, where σ is the standard deviation of posttest scores. The standard deviation of the value-added assumed two values (either

equal to, or half of, the posttest standard deviation before a treatment effect was added on). One sample was generated in each cell.

1. Normally distributed ages were generated with mean zero and variance one. Age was transformed to age in months with a mean of 48 and a standard deviation of 5.5.

2. Pretest scores, $f(\text{age}) + \epsilon$ were calculated where f is a nonlinear function of age and ϵ is a normally distributed random error term. The growth function, $Y = 107 [1 - \exp(-.070 \text{ age} + 1.626)]$, was obtained by fitting a curve to the sum of sub-scale scores of the Chicago EARLY Assessment, which was administered to 5,955 children between ages 36 and 58 months as part of an ESEA Title I program evaluation.

3. Posttest scores were calculated in the same way as pretest scores, except that a random treatment effect was added and the subject was assumed to be eight months older than at pretest time. The expected growth rate equals $f(\text{age at pretest}) - f(\text{age at posttest})$ and the pretest-posttest interval, Δ , is eight months. Because of the constraint that the test score cannot exceed 107 (the maximum possible score or ceiling), it was sometimes necessary to reduce the posttest score and treatment effect so that the posttest score would not be greater than 107.

4. Jackknifed estimates of growth rate (the regression coefficient of test score on age) were calculated using both ordinary least squares and weighted least squares (using age as a weight).

5. Using the method described in the appendix, jackknifed tobit estimates of growth rate were obtained. In addition, the following were also calculated for values of age at the mean and one and two standard deviations above and below: the growth rate for all observations, the growth rate for observations below the limit, and the proportion of change in Y due to nonlimit observations.

Results

Results of the tobit analyses are compared with OLS and WLS results in Table 1. Of the three types of estimates, the tobit estimates of the value-added have the smallest standard error (2.898 vs. 3.052 for WLS and 3.589 for OLS), the smallest bias (-3.306 vs. -6.291 for WLS and -4.123 for OLS), and the smallest MSE (36.555 vs. 62.139 for WLS and 57.953 for OLS). Although the tobit estimates were more accurate than those obtained using ordinary or weighted least squares, the null hypothesis that v was zero was never rejected in any of the 12 samples.

The dependence of the tobit growth rate estimates on age is illustrated in Table 2. The table contains, for values of age at its mean and at one standard deviations above and below the mean, the tobit estimate of growth rate for all cases, $\partial E(Y)/\partial X$; $\partial E(Y^*)/\partial X$, the tobit estimate of growth rate for nonlimit observations, i.e., those below the test ceiling (see Equation 3); and the proportion of overall growth rate that is due to nonlimit observations (see Equation 5). The ordinary least squares and weighted least squares

Table 1
Comparison of OLS, WLS, and TOBIT Estimates of the Value-Added

sample size	sample no.	% at ceiling pre/post	actual treatment effect	estimated treatment effect			std. errors of estimate			bias		
				OLS	WLS	tobit	OLS	WLS	tobit	OLS	WLS	tobit
30	1	16.7/30.0	2.282	-10.591	- 7.899	- 8.311	4.615	3.770	3.806	-12.873	-10.181	-10.593
30	2	10.0/36.7	-5.145	- 1.759	- 7.867	- 2.131	5.810	4.915	4.229	3.386	- 2.722	3.014
30	3	20.0/30.0	2.658	0.156	- 3.539	- 1.029	5.300	4.350	3.646	- 2.502	- 6.197	- 3.687
30	4	10.0/33.3	-0.872	- 6.464	-10.732	- 5.932	4.284	4.238	4.728	- 5.592	- 9.869	- 5.060
30	5	10.0/43.3	6.017	0.552	1.597	3.870	4.204	3.724	3.388	- 5.465	- 7.614	- 2.147
30	6	20.0/43.3	3.565	-10.636	- 9.908	- 6.637	3.791	3.614	3.916	-14.201	-13.478	-10.203
120	7	10.0/36.7	0.064	1.796	- 1.894	1.588	2.579	2.026	1.787	1.732	- 1.958	1.524
120	8	11.7/42.5	-0.520	- 6.969	- 7.075	- 5.920	2.439	1.976	1.949	- 6.176	- 6.555	- 5.400
120	9	8.3/34.2	2.957	2.341	- 0.088	2.660	2.533	1.958	1.629	0.616	- 3.045	- 0.298
120	10	14.2/45.8	1.834	- 1.830	- 4.476	- 1.182	2.510	2.071	1.890	- 3.664	- 6.310	- 3.016
120	11	7.5/44.2	4.278	- 0.112	- 1.590	0.609	2.624	1.941	1.842	- 4.390	- 5.868	- 3.669
120	12	12.5/51.7	2.850	2.504	1.143	2.710	2.373	2.037	1.963	- 0.346	- 1.707	- 0.140
mean							3.589	3.052	2.898	- 4.123	- 6.291	- 3.306

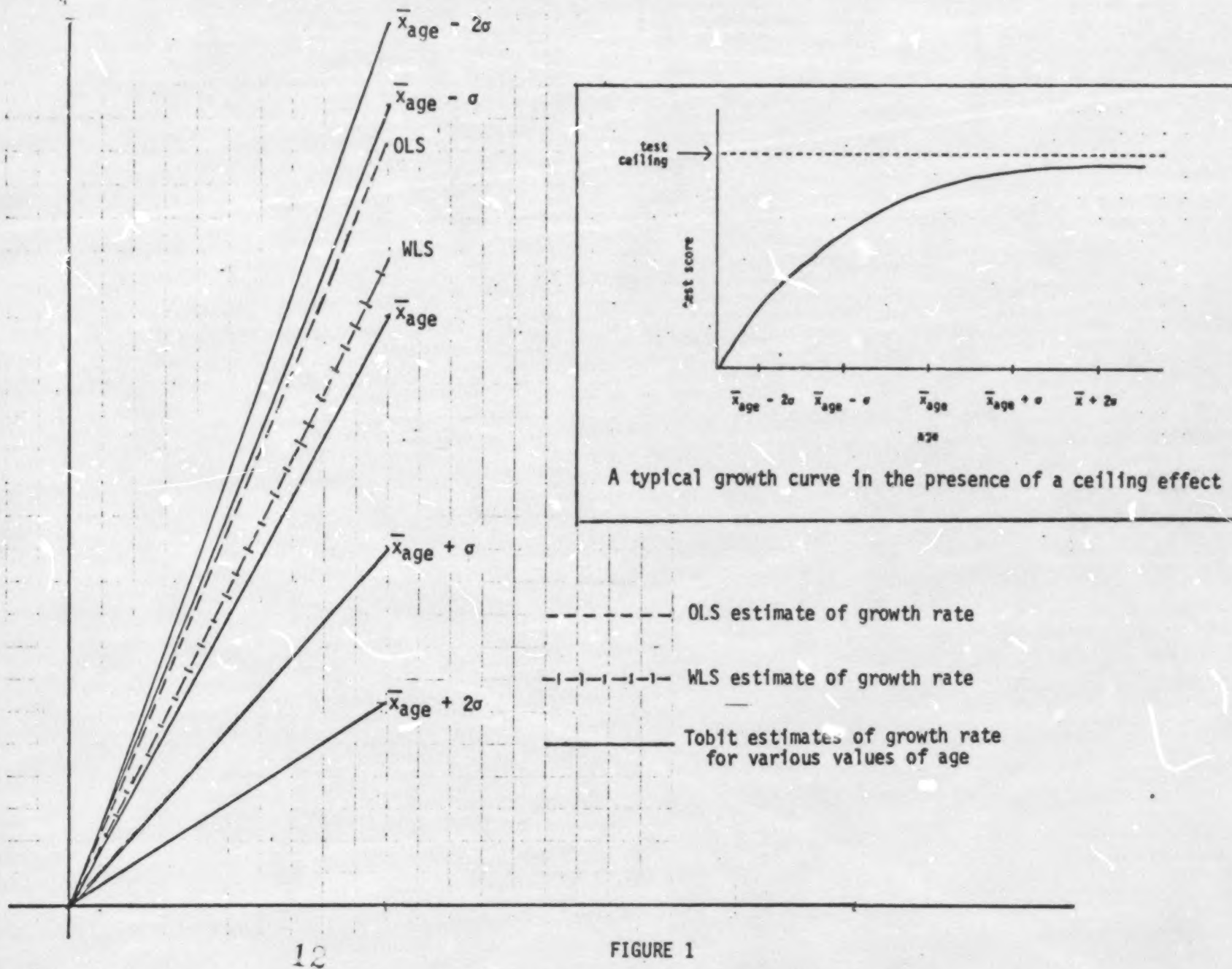
Table 1 (Cont'd) - Comparison of OLS, WLS and TOBIT Estimates of Value-Added

sample size	sample number	mean squared error			<u>t</u>			OLS growth rate	WLS growth rate	tobit growth rate at mean	tobit beta
		OLS	WLS	tobit	OLS	WLS	tobit				
30	1	187.015	117.869	126.703	-2.29	-2.10	-2.18	2.322	1.977	2.361	2.704
30	2	45.223	31.566	26.967	-0.30	-1.60	-0.50	9.346	1.094	0.401	0.419
30	3	34.346	57.313	26.890	0.03	-0.81	-0.28	0.871	1.315	0.887	1.207
30	4	49.624	115.182	47.967	-1.51	-2.53	-1.25	0.776	1.316	0.884	0.978
30	5	47.534	71.842	16.089	0.13	-0.43	1.14	1.427	1.662	1.515	1.536
30	6	216.052	194.593	119.425	-2.81	-2.74	-1.70	2.012	1.924	2.046	2.403
120	7	9.655	7.940	5.516	0.70	-0.94	0.89	0.780	1.247	0.770	0.841
120	8	47.548	46.872	32.955	-2.86	3.58	-3.04	1.662	1.679	1.630	1.839
120	9	6.797	13.111	2.743	0.92	-0.04	1.63	1.133	1.437	1.163	1.235
120	10	19.725	44.108	12.670	-0.73	-2.16	-0.63	1.076	1.410	1.105	1.262
120	11	26.162	38.204	16.857	-0.04	-0.82	0.33	1.312	1.501	1.283	1.437
120	12	5.753	7.064	3.874	1.05	0.56	1.38	1.298	1.474	1.272	1.384
mean		57.953	62.139	36.555							

Table 2

Tobit Estimates of Growth Rate for Various Values of Age

sample size	sample number	non-tobit estimates of growth rate		tobit estimate of growth rate for all cases					tobit estimate of growth rate for nonlimit observations					proportion of overall growth rate due to nonlimit observations				
		OLS	WLS	$\bar{x}-2\sigma$	$\bar{x}-\sigma$	$\bar{x}_{age}$	$\bar{x}+\sigma$	$\bar{x}+2\sigma$	$\bar{x}-2\sigma$	$\bar{x}-\sigma$	$\bar{x}_{age}$	$\bar{x}+\sigma$	$\bar{x}+2\sigma$	$\bar{x}-2\sigma$	$\bar{x}-\sigma$	$\bar{x}_{age}$	$\bar{x}+\sigma$	$\bar{x}+2\sigma$
30	1	2.332	1.977	2.700	2.653	2.361	1.567	0.620	2.670	2.436	1.815	1.106	0.628	.988	.901	.671	.409	.232
30	2	0.346	1.094	0.412	0.407	0.401	0.392	0.381	0.378	0.364	0.346	0.326	0.305	.902	.867	.826	.778	.726
30	3	0.871	1.315	0.984	0.946	0.887	0.802	0.695	0.853	0.769	0.675	0.579	0.488	.830	.749	.657	.564	.475
30	4	0.776	1.316	0.964	0.938	0.884	0.788	0.650	0.897	0.815	0.702	0.577	0.456	.917	.833	.718	.590	.466
30	5	1.427	1.662	1.536	1.535	1.515	1.371	0.926	1.535	1.524	1.417	1.076	0.651	1.000	.992	.923	.700	.424
30	6	2.012	1.924	2.394	2.327	2.046	1.418	0.670	2.330	2.062	1.543	1.000	0.612	.970	.858	.642	.416	.255
120	7	0.780	1.247	0.821	0.802	0.770	0.721	0.655	0.747	0.688	0.621	0.547	0.472	.803	.818	.738	.651	.561
120	8	1.662	1.679	1.824	1.773	1.630	1.341	0.929	1.739	1.558	1.270	0.950	0.673	.946	.847	.691	.517	.366
120	9	1.133	1.437	1.226	1.207	1.116	1.075	0.932	1.174	1.097	0.976	0.825	0.666	.951	.888	.791	.668	.539
120	10	1.076	1.410	1.234	1.190	1.105	0.967	0.790	1.121	1.002	0.852	0.694	0.548	.888	.794	.676	.550	.434
120	11	1.312	1.501	1.419	1.378	1.283	1.111	0.864	1.327	1.194	1.007	0.799	0.608	.923	.831	.700	.556	.423
120	12	1.298	1.474	1.372	1.343	1.272	1.131	0.913	1.305	1.197	1.030	0.832	0.640	.943	.864	.744	.601	.463



estimates of growth rate are also given for the purpose of comparison. While the OLS and WLS estimates of growth rate are constant across all values of ages, the tobit estimates vary considerably, becoming smaller as age increases. For the first sample, the tobit estimates of growth rate range from 2.700 at two standard deviations below the mean of age to 2.361 at the mean and 0.620 at two standard deviations above the mean. This is in contrast to the constant OLS and WLS growth rate estimates of 2.322 and 1.977, respectively. These growth rates are displayed graphically in Figure 1. Note that the tobit growth rates more closely approximate an actual growth curve than do the single growth rates obtained using OLS or WLS.

Like the overall growth rate estimates, the tobit estimates of growth rate for nonlimit observations show a decline as age increases. The decline is reasonable, since younger children tend to score lower (i.e., farther from the test ceiling) than older children and therefore, have more room to grow. This is evidenced by the fact that the proportion of the total growth rate due to nonlimit observations decreases substantially as age increases. In the example discussed above, that proportion decreases from .988 at two standard deviations below the mean to .232 at two standard deviations above the mean. The sensitivity of tobit growth rate estimates to age helps to provide more accurate estimates of the value-added than one could obtain using ordinary or weighted least squares.

Discussion

The results of this study indicate that nonlinearity caused by floor or ceiling effects can result in serious problems in treatment effect estimation. Among these problems are bias and inefficient estimates. Weighted least squares is not a panacea; in this study, it increased efficiency, but did so at the cost of increasing bias.

The usual rationale for the use of WLS is that it gives greater weight to observations at levels of the independent variable where there is relatively small variation in the dependent variable, with the implication that those observations are somehow more reliable than the others. In this study, the reduced variance is most likely the result of students clustering at the ceiling. There is no reason to believe that measurement at the test ceiling is more accurate than it is elsewhere; in fact, probably the opposite is true, especially for high-achieving students. Consequently, putting more weight on the ceiling observations served to increase the bias of the estimated treatment effect.

Tobit analysis provided more accurate estimates of the value-added than did ordinary or weighted least squares. This happened because the tobit-derived estimates of growth rate took into account that (1) there was a test ceiling and (2) growth rate depended on age (since older subjects had expected test scores relatively nearer the ceiling than younger subjects, the older subjects had less room to grow, and therefore, a lower expected growth rate).

Because it does not directly use pretest score to predict a given subject's posttest score, the value-added model cannot fully exploit some of the unique features of tobit analysis: a means of calculating the expected value for non-limit observations and the rate of change (growth rate in this context) for non-limit observations. It is also possible to determine what proportion in the overall change in $E(Y)$ with respect to X is the result of nonlimit observations.

Many different types of measures used in the social sciences have either upper or lower limits (e.g., tests, attitude surveys, data on incidence of crimes), and there are times when many observations cluster at either the maximum or minimum. In those situations, tobit analysis would seem to provide a superior alternative to more commonly used regression techniques. Increased use of tobit analysis could result in more accurate mathematical representations of educational and social phenomena. These would in turn provide a better basis for making policy decisions.

APPENDIX Method for Obtaining Jackknifed Tobit Estimates

1. For each of the 12 samples, a tobit analysis was run on the entire data set (N = 30 or 120) using the tobit subroutine from the B34S package (Stokes, 1982).
2. N individual tobit analyses were done, each omitting a single observation. These yielded the arrays BETA, CONST, and SEREG. Expected growth is defined as the difference in the expected values of Y at pretest and posttest times (see Equation 13). Let $E(Y|z) = Y_{lim} + \sigma z[1 - F(z)] - \sigma f(z)$, where

$$z_{pre} = [(BETA)(AGE_{pre}) + CONST - Y_{lim}]/SEREG$$

and

$$z_{post} = [(BETA)AGE_{pre} + t) + CONST - Y_{lim}]/SEREG.$$

In this study, the time interval, t , is eight months and the limit value or test ceiling is 107. $F(z)$ and $f(z)$ are the standard normal distribution and density functions, respectively and σ is the standard error of the tobit regression. Expected growth is then

$$\begin{aligned} & E(Y|z_{post}) - E(Y|z_{pre}) \\ &= \{Y_{lim} + \sigma z_{post}[1 - F(z_{post})] - \sigma f(z_{post})\} \\ & \quad - \{Y_{lim} + \sigma z_{pre}[1 - F(z_{pre})] - \sigma f(z_{pre})\} \\ &= \sigma \{z_{post}[1 - F(z_{post})] - f(z_{post}) - z_{pre}[1 - F(z_{pre})] + f(z_{pre})\}. \end{aligned}$$

The standard normal distribution function $F(z)$ was evaluated using the IMSL subroutine MDNOR. The density function $f(z)$ was calculated directly.

3. An overall mean gain, GAINX, was calculated by averaging the N individual gains obtained using the whole-group tobit results (BETAX, CONSTX, SEREGX).

4. An additional N mean gain estimates (the array GAINMN) were obtained using the results from step 3. Each of the elements in GAINMN was based on N - 1 observations.

5. N jackknifed expected gain estimates were calculated, where

$$JKGAIN_1 = [(N)(GAINX)] - [(N - 1)(GAINMN_1)].$$

6. Each of N pseudo-values was then calculated by subtracting the pretest score and jackknifed expected gain estimate from the posttest score. The mean of the pseudo-values was the estimated value-added.

7. The following were also calculated for values of age at the mean and one and two standard deviations above and below: $\partial E(Y)/\partial X$, the growth rate for all observations; $\partial E(Y^*)/\partial X$, the growth rate for observations below the limit; and the proportion of change in Y due to nonlimit observations.

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